Available online at www.sciencedirect.com
SCIENCE DIRECT®

Journal of Financial Intermediation 13 (2004) 265–283

Journal of Financial Intermediation

www.elsevier.com/locate/jfi

The empirical relationship between average asset correlation, firm probability of default, and asset size

Jose A. Lopez

Economic Research Department, Federal Reserve Bank of San Francisco, 101 Market Street, San Francisco, CA 94705, USA

Received 1 January 2002; revised 1 May 2003

Abstract

The asymptotic single risk factor approach is a framework for determining regulatory capital charges for credit risk, and it has become an integral part of the second Basel Accord. Within this approach, a key parameter is the average asset correlation. We examine the empirical relationship between this parameter, firm probability of default and firm asset size measured by the book value of assets. Using data from year-end 2000, credit portfolios consisting of US, Japanese, and European firms are analyzed. The empirical results suggest that average asset correlation is a decreasing function of probability of default and an increasing function of asset size. The results suggest that these factors may need to be accounted for in the final calculation of regulatory capital requirements for credit risk.

JEL classification: G20; G28

Keywords: Credit risk; Basel II; Average asset correlation; Probability of default

1. Introduction

As discussed by Gordy (2000a, 2000b, 2002), the asymptotic single risk factor (ASRF) approach is a general framework for determining regulatory capital requirements for credit risk, and it has become an integral part of the second Basel Accord. A key variable in the ASRF approach is the correlation of a given firm's assets with the risk factor that summarizes general economic conditions. In economic capital calculations, every obligor would have a unique asset correlation.

E-mail address: jose.a.lopez@sf.frb.org.

1042-9573/$ – see front matter
doi:10.1016/S1042-9573(03)00045-7

However, for the purposes of regulatory capital calculations, such a multitude of parameters is infeasible. Instead, it was originally proposed that an average correlation be used for every obligor. Specifically, in the Basel Committee on Banking Supervision (BCBS) document of January 2001 (BCBS, 2001a), asset correlations were assigned a value of 0.20 for all obligors. That is, the asset values of every obligor were assumed to have a factor loading of 0.20 with the common risk factor. In response to practitioner feedback and its own quantitative impact studies, the BCBS proposed an alternative formula in November 2001 that would make asset correlations a decreasing function of firm probability of default; see BCBS (2001c).[1]

Since the BCBS has decided that asset correlations should not be common across all obligors, the question arises as to whether there are patterns in the average asset correlation variable that are tied to other variables. The purpose of this paper is to investigate this question empirically. To do so, we imposed the ASRF approach on the KMV methodology for determining credit risk capital charges.[2] We then examined the relationship between portfolios' average asset correlations, firms' probabilities of default and firms' asset sizes as measured by the book value of assets. The analysis was conducted on portfolios of US, Japanese, and European firms, as well as on "world" portfolios consisting of firms from all of these countries, using data from year-end 2000. Subportfolios based on either firm default probability or asset size were constructed for the univariate analysis, and subportfolios based on both variables were used for the bivariate analysis.

Our empirical results confirm that average asset correlation is a decreasing function of the probability of default, as suggested by BCBS (2001c, 2002). This univariate result suggests that firms with higher default probabilities are mainly subject to idiosyncratic risks and are not tied as closely to the general economic environment summarized by the common risk factor. This result is in line with the finding by Das et al. (2002) that firms with higher credit ratings (i.e., lower PDs) are more exposed to an economy-wide factor than firms with lower credit quality. In a direct comparison, the calibrated average asset correlations derived here generally match those derived from the formula presented in the November 2001 BCBS proposal.

Our empirical results further indicate that average asset correlation is an increasing function of firm asset size. That is, as firms increase the book value of their assets, they become more correlated with the general economic environment and the common risk factor. The intuition for this result is that larger firms can generally be viewed as portfolios of smaller firms, and such portfolios would be relatively more sensitive to common risks than to idiosyncratic risks.

[1] Since the completion of this research, the BCBS has proposed another formula for asset correlation based on firm probability of default and total annual sales; see BCBS (2002) for further details.

[2] The software firm KMV, LLC is a leader in the field of credit risk modeling and capital budgeting. This study was conducted using their Portfolio Manager™ software, which they kindly provided for this study. Note that the analysis was conducted using Portfolio Manager, version 1.4.7 and prior to the release of Portfolio Manager, version 2.0 in 2002. Since the empirical results in the paper are based solely on the KMV methodology, they are highly dependent on its underlying structure. However, as shown by Gordy (2000a), several credit risks models share similar mathematical structures.

The bivariate results support both sets of univariate results and appear to highlight an additional and potentially important relationship between the three variables. The results indicate that the decreasing relationship between average asset correlation and default probability is more pronounced for larger firms. In other words, the average asset correlation for larger firms is more sensitive to firm probability of default than it is for smaller firms, probably because the average asset correlations for larger firms start at a higher level than for smaller firms. In direct comparison with the regulatory average asset correlations derived from the November proposal, the greatest deviations from the calibrated values are for portfolios composed of the largest firms, suggesting the value of incorporating firm size into the regulatory formula for average asset correlations. In fact, we find that incorporating firm size into the average asset correlation might increase regulatory capital requirements.

These results provide suggestive evidence that both firm probability of default and firm size impact average asset correlation within an ASRF framework. Hence, further work regarding whether regulatory capital requirements should take further account of such firm-specific factors seems warranted.

The paper is organized as follows. Section 2 summarizes the KMV methodology for credit risk modeling, describes how the ASRF framework was imposed onto it, and presents the credit portfolios used in the analysis. Section 3 presents the calibrated empirical results. Section 4 presents the comparison of the calibrated average asset correlations with those derived from the November 2001 regulatory formula, and Section 5 concludes.

2. Methodology

2.1. Summary of the KMV methodology

The theoretical core of the KMV methodology for evaluating credit risk is the Merton model of a firm's stock as an option on the underlying assets; see Crosbie and Bohn (2001). To measure the credit risk of a loan to a firm, the KMV methodology models the distribution of the firm's asset value over the chosen planning horizon and the corresponding distribution of the loan's value. This loan value distribution explicitly accounts for default and changes in firm credit quality.

In general, the KMV methodology, as implemented in their Portfolio Manager™ (PM) software, models the value of the assets of firm i, denoted as A_{it}, at a given horizon. At the horizon H of interest, $A_{it+\mathrm{H}}$ is modeled as

$$\ln(A_{it+\mathrm{H}}) = \ln(A_{it}) + \left(\mu_i - \frac{\sigma_i^2}{2}\right)H + \sigma_i\sqrt{H}\,\varepsilon_{it+\mathrm{H}}, \tag{1}$$

where μ_i is the asset value drift term (typically positive), σ_i^2 is the firm's asset volatility, and the firm-specific error term $\varepsilon_{it+\mathrm{H}}$ is a weighted average of common (or systematic) random factors and an idiosyncratic random factor. That is, $\varepsilon_{it+\mathrm{H}}$ is modeled as

$$\varepsilon_{it+\mathrm{H}} = R_i\xi_{it+\mathrm{H}} + \sqrt{1 - R_i^2}\,\upsilon_{it+\mathrm{H}}, \tag{2}$$

where R_i^2 measures the percentage of the firm's asset return variance attributable to the common risk factors affecting the firm, $\xi_{it+\mathrm{H}}$ is a random variable representing the firm's H-period-ahead composite risk factor, and $\nu_{it+\mathrm{H}}$ is an idiosyncratic random variable. Both $\xi_{it+\mathrm{H}}$ and $\nu_{it+\mathrm{H}}$ are assumed to have a standard normal distribution, ensuring that this decomposition of $\varepsilon_{it+\mathrm{H}}$ is also standard normal with a variance of one. Note that the R_i^2 is referred to as the asset correlation for firm i.

For each firm of interest, KMV determines its overall liabilities at time $t + H$. For a particular realization of future asset value $A_{it+\mathrm{H}}$, a "distance to default" measure is calculated and used to determine a firm's "expected default frequency™" (or EDF™) based on KMV's proprietary default database. The firm's EDF value and its loans' recovery rates permit the calculation of the loan values if the firm defaults. For non-default states, the loans are valued by discounting their cashflows using market-based credit spreads corresponding to the firm's credit quality. Individual loan calculations can be aggregated up to the portfolio level in order to determine the distribution of loan portfolio returns, which in turn can be used for determining economic capital allocations and other credit risk management calculations.

2.2. Application of the ASRF framework to the KMV methodology

In order to establish regulatory capital requirements applicable across institutions and across credit risk models, the regulatory proposals considered by the BCBS are based on a very general modeling framework, commonly known as the asymptotic single risk factor (ASRF) approach. As described by Gordy (2000a, 2000b, 2002), the ASRF approach assumes that a single risk factor is responsible for credit quality movements across all obligors in an infinitely granular portfolio.[3] Each obligor has a unique asset correlation R_i^2 with the common risk factor, and the realization of this factor determines the obligors' individual outcomes. Further simplifying assumptions are often made to reduce the number of parameters; for example, all obligors are assumed to have a common (or average) asset correlation with the risk factor; i.e., $R_i^2 = R^2$, $\forall i$. Within this analytic framework, the regulatory capital requirement for a portfolio equals the sum of the regulatory capital requirements for the individual credits. This additive property permits the "bucketing" of credits based on certain characteristics, such as firm default probability and recovery rates. This property of the ASRF approach clearly simplifies the allocation of regulatory capital.

To impose the ASRF approach within the KMV methodology, four key restrictions were imposed on the PM software. The first and most important restriction was imposing a single risk factor across all firms; i.e., $\xi_{it+\mathrm{H}} = \xi_{t+H}$, $\forall i$. Within the KMV methodology, there are about 110 factors based on global, regional, country, sector and industry effects. These various factors are aggregated based on firm characteristics to construct a firm's composite factor. For example, the return on the composite factor (denoted CF_i) for a US domestic

[3] The impact of the assumption of infinitely granular portfolios was not part of our analysis to date. In our analysis, we imposed the assumption of common loan sizes across the obligors. We further assumed that the numbers of obligors in our sample portfolios were sufficiently large for this assumption to hold. Further research into the validity of these assumptions is necessary.

firm that is 60% involved in paper production and 40% in lumber production is

$$CF_i = 1.0 r_{\text{USA}} + 0.6 r_{\text{paper}} + 0.4 r_{\text{lumber}}, \tag{3}$$

where r_{USA}, r_{paper} and r_{lumber} are the returns on the factors for the entire US economy, the global paper industry and the global lumber industry, respectively. A regression of firm i's asset returns on CF_i provides the asset correlation R_i^2 used in their asset value simulations.

Thus, to impose the ASRF approach, we collapsed the many potential factors into a single factor common to all obligors in the credit portfolios. This restriction is imposed by forcing all of the firms to have the same degree of dependence on the same country and industry factors. In our analysis, we assumed that all obligors were dependent on the US country factor (regardless of their country of origin) and on the unassigned industry factor known as N57 within the KMV industry database.[4]

The second key restriction is the imposition of a common degree of dependence by assuming a common R^2 value for all obligors. This common R^2 value is termed the average asset correlation, even though it is not strictly an average, and it will be denoted here as $\bar{\rho}_{\text{A}}$. These two restrictions are obviously quite strong, but necessary for applying the ASRF framework.

There is no theoretical answer as to what the value of the average asset correlation should be, and thus empirical values must be determined. To calibrate the empirical value of $\bar{\rho}_{\text{A}}$ for a portfolio at a specified loss tail quantile, we minimized the absolute difference between the credit losses indicated by the unconstrained PM model and by the ASRF-constrained version. The calibrations were conducted using a grid search over a reasonable range for $\bar{\rho}_{\text{A}}$ values. Note that the relationship between this difference of credit losses and $\bar{\rho}_{\text{A}}$ values is roughly in linear due to the standard normal distribution underlying the single common risk factor $\xi_{t+\text{H}}$. The convergence criteria used were that the calibrated $\bar{\rho}_{\text{A}}$ values would only have up to four significant digits and that the dollar differences between the two models' capital charges at the specified quantile were less than 0.1% of the total portfolio size. The calibrated $\bar{\rho}_{\text{A}}$ correlations for credit portfolios composed of firms with similar default probabilities, asset sizes and national origin are the core contribution of this paper.

The third restriction needed to impose the ASRF framework within the PM software is that the recovery rate is constant. For the sake of simplifying our analysis, we imposed a recovery rate of 50% across all obligors. Although this is a strong assumption, it is a commonly used value, as in Jackson et al. (2003).[5] Furthermore, we know that capital charges within the PM software for portfolios with other common recovery rates are a simple multiple of each other.

The fourth restriction imposed on the PM software for our analysis was a one-year maturity for all credits. This restriction is not explicitly required by the ASRF framework. In

[4] Within the ASRF framework, the recommended capital requirements are independent of the common factor chosen. However, in the empirical implementation, differences will arise when different factor specifications are used. These differences should be minor, and preliminary results support this hypothesis.

[5] Jackson et al. (2003) argue that an LGD value of 50% is close to the recovery rate of 51% commonly used for senior unsecured bond defaults. Carty and Lieberman (1996) found that the recovery rates for senior secured bank loans, senior secured bonds, senior unsecured bonds, and subordinated bonds are 71%, 57%, 46%, and 34%, respectively. Asarnow and Edwards (1996) found a 65% average recovery rate for loans. However, Carey (2002) uses an LGD value of 30%.

fact, the current average maturity assumed in BCBS (2002) is three years. Further research on the impact of maturity on the calibrated $\bar{\rho}_A$ values derived within the ASRF approach is needed.

2.3. The procedure for generating and analyzing the average asset correlations

The procedure for implementing the ASRF approach within the PM software and analyzing the calibrated average asset correlations consists of three steps. The first step is to create the portfolios of interest based on firm EDF values, firm asset sizes or both. This step is described in detail in the next section. Loans to the chosen firms are then structured to have a maturity of one year, a floating rate coupon with a quarterly payment schedule and a common commitment size of $100 million. These restrictions obviously impact the nature of the credit portfolios being analyzed; for example, the standard commitment size precludes analysis of the granularity issue. Further research into this issue is required. However, the empirical results should provide meaningful insight into the relationships of interest.

The second step consists of running the unconstrained version of the PM software on the constructed portfolios in order to generate the capital required at the one-year horizon for the 99.9% percentiles of the loss distributions.[6] Of the various definitions of capital used in the PM software, we chose to use capital in excess of expected loss. That is, PM generates the portfolio's credit loss distribution, designates its mean as the expected loss, and presents the tail quantiles as credit losses beyond the expected loss. Credit losses at the one-year horizon are transformed into capital charges by discounting them to the present with the appropriate risk-free rate. Total capital is the sum of the discounted expect loss and specified tail loss.

The third step is to calibrate the portfolios' $\bar{\rho}_A$ values using a grid search as previously described. The results for the aggregate portfolios, portfolios based on EDF categories, portfolios based on size categories and portfolios based on both variables are then analyzed.

2.4. Credit portfolios of interest

For this study, we constructed national credit portfolios of US, Japanese, and European firms. The European portfolios consisted of firms based in Britain, France, Germany, Italy, and the Netherlands. We also examined aggregate (or "world") portfolios that included all of the firms in the national portfolios. Aside from the question of firms' national origins, the construction of these credit portfolios required a balance between two criteria. The first criterion was constructing portfolios that had EDF and asset size ranges that were of economic interest. The second criterion was insuring that these portfolios had a sufficient number of credits to diminish small sample concerns.

With respect to the EDFs, the portfolios were basically constructed using the S&P rating categories set within KMV's Credit Monitor© database as of year-end 2000. The rating

[6] The number of simulations that we typically used in our analysis was 100,000 runs, which is the number recommended by KMV for analysis of the 99.9% tail quantile.

categories were collapsed into five EDF categories and held constant for our analysis. The first EDF category ranged from zero to a 0.05% default probability and corresponded to AAA and AA-rated firms; the second EDF category ranged from 0.05% to 0.52% and corresponded to A and BBB-rated firms; the third EDF category ranged from 0.52% to 2.03% and corresponded to BB-rated firms; the fourth EDF category ranged from 2.03% to 6.94% and corresponded to B-rated firms; and the fifth EDF category ranged from 6.94% to 20%, the maximum EDF value permitted, and corresponded to the remaining C and D-rated firms.[7]

With respect to asset size, the economic criterion for constructing portfolios was less meaningful. Hence, we chose size categories that were sufficiently different to ensure that we captured important relative size differences, but we erred on the side of making the portfolios large enough to provide meaningful empirical results. Note that the asset size categories could not be held constant across the national groups in a meaningful way.[8]

With respect to the portfolios based on both EDF and size categories used in the bivariate analysis, the challenge was to create portfolios whose numbers of credits were approximately uniformly distributed across both sets of categories. However, this task proved impossible given the general dearth of small firms with low EDF values and large firms with high EDF values. Thus, for the bivariate analysis, we collapsed the previous five categories per characteristic into three categories, which generated nine EDF and asset size portfolios for each set of firms. We used the same three EDF categories across the sets of firms, and they were (0%, 0.52%], [0.52%, 6.94%] and [6.94%, 20.00%]. However, the size categories again varied across the country groupings.

3. Calibration results

The calibrated $\bar{\rho}_A$ values are presented below in four subsections corresponding to the aggregate portfolios, univariate analysis of the portfolios constructed using EDF categories, univariate analysis of the portfolios constructed using asset size categories, and bivariate analysis of the portfolios constructed using both EDF and asset size categories. The empirical results are presented for the 99.9% tail percentiles of the credit loss distributions.

3.1. Calibration results for the aggregate portfolios

As discussed above, we constructed three sets of firms based on country of origin, as well as the superset of these firms, in order to conduct our analysis. The calibrated $\bar{\rho}_A$ values for these portfolios are summarized in Table 1.

For the "world" portfolio, a total of 13,839 firms were used in the exercise with about 50%, 24%, and 26% being of US, Japanese, and European origin, respectively. The cali-

[7] Note that in constructing portfolios based on EDF ranges, firms with EDF values at the ends of the ranges are included in both portfolios. For example, portfolios constructed using the second and third EDF categories have in common all firms with an EDF value exactly equal to 0.52%. The number of firms in two EDF categories is small, but non-zero.

[8] The exchange rates used were as of year-end 2000.

Table 1
Calibrated average asset correlations for aggregate portfolios

Portfolio	Number of firms	Calibrated average asset correlations	
		99.5% percentile	99.9% percentile
All firms	13,839	0.1125	0.1125
US firms	6909	0.1625	0.1625
Japanese firms	3255	0.2625	0.2625
European firms	3675	0.1250	0.1375

Notes. The portfolio of all firms contains all of the US, Japanese, and European firms in the KMV CreditMonitor database as of year-end 2000. European firms are defined here as firms headquartered in France, Germany, Great Britain, Italy, and the Netherlands. Note that US, Japanese, and European firms make up 50%, 24%, and 26%, respectively, of all the firms examined here.

brated $\bar{\rho}_{\mathrm{A}}$ value for this portfolio was relatively low at 0.1125, potentially suggesting that a single risk factor is not sufficient to capture the heterogeneous nature of the credits in the portfolio.[9]

This possibility seems to be reinforced by the differing $\bar{\rho}_{\mathrm{A}}$ values for the three national portfolios. At the 99.9% percentile, these values are 0.1625 for the US portfolio, 0.2625 for the Japanese portfolio and 0.1375 for the European portfolio. The Japanese value is noticeably higher than the others, probably due to the generally poor economic conditions within Japan at year-end 2000. These conditions most likely impacted all Japanese firms in a similar way. The European value is relatively low, suggesting that a single factor may not be sufficient to capture the heterogeneity across the five countries that compose that portfolio.

The differing values suggest that the degree of heterogeneity across borrowers' countries of origin may not be well captured by a single risk factor. However, this difficulty is endemic to the ASRF approach. As discussed by Gordy (2002), the use of a single risk factor imposes a common business cycle on all obligors, and all other elements of credit risk are considered to be idiosyncratic. Under this construct, the average asset correlation for a heterogeneous loan portfolio, such as a collection of firms from a cross-section of countries, should be lower than for a homogeneous portfolio with similar characteristics.

3.2. Univariate calibration results for portfolios based on EDF categories

Within the ASRF framework, we expect to see a negative relationship between firms' asset correlations and their probabilities of default (PD). That is, as a firm's PD increases due to its worsening condition and approaching possible default, it is reasonable to assume that idiosyncratic factors begin to take on a more important role relative to the common, systematic risk factor. The calibrated $\bar{\rho}_{\mathrm{A}}$ value for the portfolios based on EDF categories support this intuition, as shown in Table 2.

For the world portfolios consisting of all firms, the calibration results indicate that the calibrated $\bar{\rho}_{\mathrm{A}}$ values decline, if only gradually, as the EDF values of the categories increase.

9 Note that Varotto (2000) found an average asset correlation between 0.09 and 0.10 for a large database of euro-bond returns.

Table 2
Calibrated average asset correlations for portfolios based on EDF categories

EDF categories (%)	Calibrated average asset correlations at the 99.9% percentile			
	World portfolios	US portfolios	Japanese portfolios	European portfolios
(0.00, 0.05]	0.1500	0.2250	0.3250	0.1750
[0.05, 0.52]	0.1500	0.2500	0.3250	0.1750
[0.52, 2.03]	0.1250	0.2000	0.2750	0.1500
[2.03, 6.94]	0.1125	0.1750	0.2500	0.1250
[6.94, 20.00]	0.1000	0.1500	0.2750	0.1250

Note. The EDF intervals are overlapping at their endpoints. This overlap leads firms with EDF values of exactly 0.05%, 0.52%, 2.03%, and 6.94% to be included in two different portfolios each.

For the lowest category, the calibrated $\bar{\rho}_{\mathrm{A}}$ value is 0.1500, and for the highest category, it is 0.1000. However, the national portfolios show more variation in this pattern.

For the US portfolios, the calibrated $\bar{\rho}_{\mathrm{A}}$ values are higher than for the world portfolios, and the slope of the decline is steeper. The value for the lowest EDF category is 0.2250, and the value for the highest category is 0.1500.[10] That is, the decline from the lowest to the highest EDF categories is 0.0750, as opposed to 0.0500 for the world portfolios. A possible reason for this result is that the world portfolios' greater degree of heterogeneity lessens the impact of the EDF decline on the calibrated $\bar{\rho}_{\mathrm{A}}$ values.

The Japanese portfolios vary the most from the other national portfolios in that the calibrated $\bar{\rho}_{\mathrm{A}}$ values are higher across all EDF categories, ranging from 0.3250 to 0.2750. The decline of just 0.0500 across the categories is in line with the world and European portfolios, but the higher levels imply a smaller percentage decline than for the other national portfolios. As mentioned before, the state of the Japanese economy at year-end 2000 was such that less firm heterogeneity in condition was possible, and this circumstance might be well explained by a single factor, even across the EDF categories.

In contrast, the European portfolios have the lowest calibrated $\bar{\rho}_{\mathrm{A}}$ values, ranging from 0.1750 to 0.1250. These lower values are in line with the world portfolio, suggesting that the heterogeneity among the European firms in the sample is also hard to capture with a single, common risk factor.

3.3. *Univariate calibration results for portfolios based on asset size categories*

In the general theory of portfolio diversification, as the number of different securities within a portfolio increases, the portfolio becomes more diversified, and the idiosyncratic element of the portfolio's return becomes less important. An analogous view could be taken with respect to a firm's asset size; that is, as a firm becomes larger and comes to contain more assets, its risk and return characteristics should more closely resemble the overall asset market and be less dependent on the idiosyncratic elements of the individual business lines. Within an ASRF framework, this intuition suggests that a firm's asset

[10] Gordy (2000a) and Hamerle et al. (2002) calibrated asset correlation values calibrated using S&P's historical default data for US firms, and their values were typically less than 0.0500. Note, however, that they did not specifically calibrate to the 99.9% tails of the credit loss distributions.

Table 3
Calibrated average asset correlations at the 99.9% percentile for portfolios based on size categories

World portfolios		US portfolios		Japanese portfolios		European portfolios	
Asset size categories	$\bar{\rho}_A$	Asset size categories	$\bar{\rho}_A$	Asset size categories	$\bar{\rho}_A$	Asset size categories	$\bar{\rho}_A$
($0, $20m]	0.1000	($0, $20m]	0.1000	($0, $100m]	0.2000	($0, $25m]	0.1125
[$20m, $100m]	0.1000	[$20m, $100m]	0.1500	[$100m, $200m]	0.2000	[$25m, $75m]	0.1125
[$100m, $300m]	0.1125	[$100m, $300m]	0.1750	[$200m, $400m]	0.2500	[$75m, $200m]	0.1250
[$300m, $1b]	0.1375	[$300m, $1b]	0.2250	[$400m, $1,b]	0.3000	[$200m, $1b]	0.1500
$\geqslant$ $1b	0.2000	$\geqslant$ $1b	0.3000	$\geqslant$ $1b	0.4500	$\geqslant$ $1b	0.2250

Notes. The asset size intervals are overlapping at their endpoints. This overlap leads firms with asset sizes at the end of an interval to be included in two different portfolios each.

correlation should increase as its asset size increases. However, this hypothesis must be verified empirically. The calibration results based on asset size categories are presented in Table 3.[11]

For the world portfolios based on all the firms in the three national portfolios, the calibrated $\bar{\rho}_A$ value doubles from 0.1000 for firms smaller than $20 million in assets to 0.2000 for firms larger than $1 billion in assets. The patterns for the national portfolios are similar, but they show certain differences.

For the US portfolio, the calibrated $\bar{\rho}_A$ values actually increase by more, but the rate of increase across the size categories is more gradual. The calibrated $\bar{\rho}_A$ value is 0.1000 for US firms smaller than $20 million and triples to 0.3000 for firms larger than $1 billion. This increase is approximately linear across the chosen size categories, while the increase for the world portfolios appears to be more exponential in nature. A potential reason for this difference appears to be that the Japanese firms in the world portfolios are generally of larger size than the US firms. Approximately 20% of the sample's Japanese firms fall into the category of less than $100 million in assets, while about 44% of US and European firms in the sample fall into these categories. Since firms with assets greater than $1 billion make up about 20% of each of the three national samples, the 24 percentage point difference for the Japanese firms is accounted for by the category of firms between $100 million and $1 billion in assets.

The calibrated $\bar{\rho}_A$ values for the Japanese portfolios are again higher than those for the other portfolios. In fact, for these size portfolios, the calibrated $\bar{\rho}_A$ values are much higher, with values of 0.2000 for firms smaller than $100 million and 0.4500 for firms larger than $1 billion, a multiple of 2.5 across the size categories. The Japanese portfolios are clearly different from the US and European portfolios, and hence they are probably contributing greatly to the shape of the increases for the world portfolio.

As before, the calibrated $\bar{\rho}_A$ values for the European firms are generally the lowest of the three aggregate portfolios, ranging from 0.1125 for firms smaller than $25 million and 0.2250 for firms larger than $1 billion in assets. The doubling of the calibrated $\bar{\rho}_A$ values

[11] Note that the asset size intervals are overlapping at their endpoints. This overlap leads firms with asset sizes at the end of an interval to be included in two different portfolios each.

across the size spectrum equals that of the world portfolios and is the lowest of the national portfolios. The pattern of increases again seems to be more exponential than linear.

3.4. Bivariate calibration results for portfolios based on EDF and asset size categories

The results discussed in the prior two sections indicate that average asset correlations within an ASRF framework appear to be a decreasing function of EDF and an increasing function of asset size. In this section, we examine the extent to which calibrated $\bar{\rho}_A$ values are a function of both variables simultaneously. We did so by forming nine portfolios based on both EDF and asset size categories for each of the four sets of firms.[12]

The calibration results presented in the panels of Table 4 suggest three key outcomes. First, as in the univariate results, the calibrated $\bar{\rho}_A$ values are generally decreasing functions of EDF and increasing functions of asset size. Thus, both of the univariate results are important to consider. Second, the increases in the calibrated $\bar{\rho}_A$ values with respect to increases in size appear to be larger than the decreases with respect to increasing EDF values. In other words, the marginal impact on a calibrated $\bar{\rho}_A$ of an increase in firm asset size is usually larger than the marginal impact of an increase in firm EDF value. Third, the changes in the calibrated $\bar{\rho}_A$ values across EDF categories appear to become greater as

Table 4
Calibrated average asset correlations at the 99.9% percentile for portfolios based on EDF and asset size categories

Asset size categories	EDF categories (%)		
	(0.00, 0.52]	[0.52, 6.94]	[6.94, 20.00]
	A. World portfolios		
($0m, $75m]	0.1000	0.1000	0.1000
[$75m, $500m]	0.1125	0.1125	0.1125
⩾ $500m	0.2000	0.1750	0.1625
	B. US portfolios		
($0m, $100m]	0.1375	0.1250	0.1250
[$100m, $1000m]	0.1875	0.1875	0.1750
⩾ $1000m	0.3000	0.2750	0.2250
	C. Japanese portfolios		
($0m, $200m]	0.2250	0.2000	0.2000
[$200m, $1000m]	0.2500	0.2500	0.2750
⩾ $1000m	0.4250	0.4000	0.5500
	D. European portfolios		
($0m, $25m]	0.1250	0.1250	0.1250
[$25m, $200m]	0.1250	0.1250	0.1250
⩾ $200m	0.2000	0.1750	0.1750

Notes. The asset size and EDF intervals are overlapping at their endpoints. This overlap leads firms with asset sizes and/or EDF values at the ends of intervals to be included in multiple portfolios.

[12] Note that as before, the EDF categories are common across the three national portfolios and the world portfolio, but the asset size categories vary. Note that the asset size and EDF intervals are overlapping at their endpoints. This overlap leads firms with asset sizes and/or EDF values at the ends of intervals to be included in multiple portfolios.

firm asset size increases. That is, as firms get larger, changes in EDF lead to larger changes in firm sensitivity to the common risk factor.

Two important caveats regarding the calibration results in this section should be noted. First, the analysis was conducted using discrete EDF and size categories and not using continuous functions. Hence, any inference on $\bar{\rho}_{\mathrm{A}}$ changes across the categories is relatively limited and cannot be easily tested. However, the general patterns of variation that are observed should provide reasonably suggestive evidence. Second, the number of firms in the portfolios varies and can be quite low in the extreme categories, such as the largest firms with highest EDF values and the smallest firms with lowest EDF values. These data limitations are endemic to this type of analysis and can effectively be seen as increasing the standard errors around the calibrated $\bar{\rho}_{\mathrm{A}}$ values for those portfolios. This larger degree of uncertainty obviously limits the inference that can be drawn from the results. However, the general patterns discussed here should be robust since they are also observed for portfolios that have reasonable numbers of observations.

Turning directly to the results for the nine world portfolios in the first panel of Table 4, the calibrated $\bar{\rho}_{\mathrm{A}}$ values for the first two size categories do not vary across the EDF categories, but they do decline for the third category, as before. The reasons for the constant $\bar{\rho}_{\mathrm{A}}$ values across the EDF categories are not clear, but this result could be indicative that firm size is a potentially more important factor in determining average asset correlations. Looking across asset size categories for a given EDF category, we observe that the calibrated $\bar{\rho}_{\mathrm{A}}$ values increase with size, as per the univariate results. For these portfolios, variation in asset size clearly has a greater impact on the calibrated $\bar{\rho}_{\mathrm{A}}$ values than variation in EDF values.

The calibrated $\bar{\rho}_{\mathrm{A}}$ values for the nine US portfolios in the second panel of Table 4 range from 0.1375 to 0.3000. Overall, these values are higher than the values for the world portfolio, further indicating that national portfolio diversification issues could impact the average asset correlations. The univariate patterns previously observed are apparent in the bivariate analysis as well. Looking across the categories, the calibrated $\bar{\rho}_{\mathrm{A}}$ values decrease from the lowest to the highest EDF categories within a given size category by roughly 0.1000, whereas the $\bar{\rho}_{\mathrm{A}}$ values approximately double from the smallest to the largest size category within a given EDF category. This result further suggests the importance of firm size in determining a portfolio's average asset correlation.

The calibrated $\bar{\rho}_{\mathrm{A}}$ values for the Japanese portfolios are presented in the third panel of Table 4. As before, the $\bar{\rho}_{\mathrm{A}}$ values for these portfolios are substantially higher than those of the other national portfolios. The values range from 0.2000 to 0.5500, although this latter value for the portfolio with the largest firms and the highest EDF values is based on a sample consisting of just 2% of the Japanese sample. Although the pattern of declining $\bar{\rho}_{\mathrm{A}}$ values within a size category due to increasing EDF values is not as apparent here, the $\bar{\rho}_{\mathrm{A}}$ values do increase with asset size for a given EDF category. In fact, the size effect is strongest for the Japanese results.

The calibrated $\bar{\rho}_{\mathrm{A}}$ values for the European portfolios in the last panel of Table 4 range from 0.1250 to 0.2000. Note that these values do not vary much across either EDF or asset size categories. In fact, $\bar{\rho}_{\mathrm{A}} = 0.1250$ for all EDF categories across the first two asset size categories. Only for the largest EDF category do we observe an increase in $\bar{\rho}_{\mathrm{A}}$ across the size categories and a decline across the EDF categories. This relative lack of $\bar{\rho}_A$ variation is approximately similar to the univariate results, but further analysis is needed here.

An alternative way of measuring the marginal impact of incorporating firm EDF and asset size in the calibrated $\bar{\rho}_A$ values is to examine the relative value of the corresponding regulatory capital charges.[13] That is, we can quantify the impact of modifying the calibrated $\bar{\rho}_A$ values for our aggregate portfolios by examining the changes in their 99.9% tail quantile losses. A priori, we would expect that as a portfolio's weighted average $\bar{\rho}_A$ across obligors increases that the implied tail loss would also increase. In comparing the calibrated $\bar{\rho}_A$ values in Table 1 with those in Tables 2 through 4, we observe that the weighted average $\bar{\rho}_A$ values increase, and we should expect that the regulatory capital charges for our aggregate portfolios to increase.

Turning first to the $\bar{\rho}_A$ values incorporating just the EDF categories in Table 2, the 99.9% tail losses increase by an average of 2.0% across the four aggregate portfolios. However, when the $\bar{\rho}_A$ values based on the asset size categories in Table 3 are used, the tail losses increase by a larger average of 5.1%. Similarly, the $\bar{\rho}_A$ values based on both categories raise the aggregate portfolios' tail losses by an average of 4.0%. These results further highlight the important effect that firm size has on the $\bar{\rho}_A$ values used in the ASRF framework.

In conclusion, the analysis of the calibrated $\bar{\rho}_A$ values using an ASRF framework within the KMV methodology provides strong suggestive evidence that EDF values and, more importantly, firm size are important driving factors. In addition, the results also suggest national characteristics are potentially important, but more analysis over other time periods is necessary to verify this result.

4. Regulatory values for average asset correlation

In an effort to update and improve international bank capital requirements, the BCBS began a revision of the 1988 Basel Accord in 1999, a process that has come to be known as "Basel II." A primary goal of the Basel II process is to make credit risk capital requirements more risk-sensitive, i.e., more closely linked to the economic risks faced by banks in their business activities. Credit risk modeling is to play an important role in this process.

As detailed in BCBS (2001a), the BCBS decided to adopt an evolutionary approach to the implementation of credit risk capital requirements, an approach that mirrors the evolution of credit risk management techniques themselves.[14] For banks that do not have sufficiently well developed credit risk models, credit risk capital requirements are to be determined using the "standardized" approach, which effectively does not require modeling input from the banks. However, for banks that are employing more quantitatively oriented techniques of credit risk management, particularly internal risk-rating systems, an internal-ratings based (IRB) approach was proposed.

The IRB approach has two tiers within it. The first tier, known as the "foundation" IRB approach, permits banks to input their own credit risk assessments of their lending portfo-

[13] I thank a referee for this suggestion.

[14] For a survey of the credit risk modeling at large financial institutions, see BCBS (1999). See Hirtle et al. (2001) for a discussion of the policy options for using such models for regulatory purposes.

lios, but estimation of additional inputs are derived through the application of standardized regulatory rules. This IRB approach is intended to be used by banks that currently face difficulty in estimating certain risk model parameters due to data limitations or less-developed credit risk models. The second tier, known as the "advanced" IRB approach, would allow banks to use their own internal credit risk modeling outcomes to establish their regulatory credit risk capital requirements.

As mentioned earlier, the analytical framework used for determining credit risk capital requirements within the Basel II process is the ASRF framework. Within this framework, as discussed in Section 2, the average asset correlation is a key parameter, but regulators need only specify a $\bar{\rho}_{\mathrm{A}}$ value for the foundation IRB approach. In paragraph 172 of the January 2001 proposal (BCBS, 2001a), the regulatory value of $\bar{\rho}_{\mathrm{A}}$ was set to be constant at 0.2000, based on surveys of industry practice and research conducted by the BCBS.

This assumption, as well as the various others that compose the IRB approach, were examined within the context of two quantitative impact studies; the second of which, known as QIS-2, was initiated in April 2001 and summarized in BCBS (2001b). In light of those results, the BCBS initiated a more targeted study of the foundation IRB approach in November 2001, which is known as QIS-2.5; see BCBS (2001c).[15] Within this study, the asset correlation assumption was modified to make ρ_{A} a function of the probability of default (PD). Specifically,

$$\rho_{\mathrm{A}}(\mathrm{PD}) = 0.1000\left(\frac{1-\mathrm{e}^{-50*\mathrm{PD}}}{1-\mathrm{e}^{-50}}\right) + 0.2000\left(1-\frac{1-\mathrm{e}^{-50*\mathrm{PD}}}{1-\mathrm{e}^{-50}}\right), \tag{4}$$

or equivalently,

$$\rho_{\mathrm{A}}(\mathrm{PD}) = 0.2000 - 0.1000\left(\frac{1-\mathrm{e}^{-50*\mathrm{PD}}}{1-\mathrm{e}^{-50}}\right). \tag{5}$$

Note that $\rho_{\mathrm{A}}(\mathrm{PD})$ is bounded within the interval [0.1, 0.2].

In this section, we further analyze our calibrated $\bar{\rho}_{\mathrm{A}}$ values with respect to both the proposed January and November regulatory $\bar{\rho}_{\mathrm{A}}$ values. Specifically, the calibrated $\bar{\rho}_{\mathrm{A}}$ values are compared to the assumption that $\bar{\rho}_{\mathrm{A}} = 0.2000$ and to the November proposal evaluated at the median EDF values of the constructed portfolios.

These comparisons suggest five key results. First, the overall comparison of the calibrated $\bar{\rho}_{\mathrm{A}}$ values and the January proposal suggests that there is sufficient variation as to make the constant assumption too inaccurate. Second, the overall comparison of the calibrated $\bar{\rho}_{\mathrm{A}}$ values and the November proposal are rather favorable, especially for higher EDF categories and more diversified portfolios. However, there are important areas of difference. Third, limiting the maximum $\rho_{\mathrm{A}}(\mathrm{PD})$ value to 0.2000 is not warranted by the calibration results. Fourth, the decline in $\rho_{\mathrm{A}}(\mathrm{PD})$ due to increases in PD values is quicker than suggested by the calibrated $\bar{\rho}_{\mathrm{A}}$ values. Fifth, the formula in the November proposal does not account for size, and the greatest deviations from the calibrated $\bar{\rho}_{\mathrm{A}}$ values are for

[15] The BCBS clearly states that it "has not at this stage endorsed the specific modifications that are the focus of the additional quantitative impact exercise." In fact, BCBS (2002), which was released after this research was completed, proposed another formulation of the asset correlation parameters.

the larger firms, further suggesting the potential value of incorporating some element of firm size into the regulatory values for average asset correlations.

4.1. Comparison for aggregate portfolios

The comparison of the calibrated $\bar{\rho}_A$ values for the four aggregate portfolios with the regulatory values is presented in Table 5. Overall, the two sets of $\bar{\rho}_A$ values are similar, with the exception of the Japanese portfolio. The calibrated $\bar{\rho}_A$ value for this portfolio is 0.2625, while the November regulatory proposal suggests a value of 0.1430, which is more in line with the value for the regulatory values of the other three portfolios. These results suggest that for broadly diversified portfolios, the November proposal for regulatory $\bar{\rho}_A$ values appears to match the calibration results quite well, although country concentrations may be a concern.

4.2. Comparison for portfolios based on EDF and asset size categories

The comparison of calibrated $\bar{\rho}_A$ values for the portfolios based on both EDF and asset size categories with the regulatory values based on the November proposal are presented in the panels of Table 6.[16] Note that the regulatory $\bar{\rho}_A$ values also increase across the asset size categories for a given EDF category because of the increasing median EDF values. Overall, the differences between the alternative measures are generally small. Of the 36 portfolios examined, the absolute difference in $\bar{\rho}_A$ values was less than 0.0500 for 47% of the cases and just 37% when the Japanese portfolios are excluded. The deviations were largest for firms in the largest size categories, which accounted for 50% of the deviations excluding the Japanese portfolios. These results suggest that the calibrated $\bar{\rho}_A$ values deviate most from the regulatory values, which do not take size into account, for the largest firms in the four firm samples.

Table 5
Comparison of average asset correlations for the aggregate portfolios

Portfolio	Calibrated avg. asset correlation	January correlation proposal	November correlation proposal
All firms	0.1125	0.2000	0.1445
US firms	0.1625	0.2000	0.1300
Japanese firms	0.2625	0.2000	0.1430
European firms	0.1375	0.2000	0.1684

Notes. The table contrasts the average asset correlations as calculated in three different ways for the aggregate portfolios. The calibrated average asset correlation is the value reported earlier with respect to the 99.9% percentile. The January correlation proposal value is the constant value of 0.20 implied in the January 2001 proposal by the Basel Committee on Banking Supervision. The November correlation proposal values are the values derived from the formula in the November 2001 proposal by the Basel Committee on Banking Supervision. The formula is presented in the text. The median EDF values for the aggregate portfolios are 1.6 for all firms, 2.4 for US firms, 1.7 for Japanese firms, and 1.8 for European firms.

[16] To conserve space, the comparison of the calibrated average asset correlations based on just EDF and just size categories are not discussed here, but the results are available from the author upon request.

Table 6
Comparison of average asset correlations for the portfolios based on EDF and asset size categories

Porfolios	Asset size ($m)	EDF (%)	Calibrated average asset corr.	January correlation proposal	November correlation proposal
A. World	(0, 75]	(0.00, 0.52]	0.1000	0.2000	0.1909
		[0.52, 6.94]	0.1000	0.2000	0.1311
		[6.94, 20.00]	0.1000	0.2000	0.1000
	[75, 500]	(0.00, 0.52]	0.1125	0.2000	0.1905
		[0.52, 6.94]	0.1125	0.2000	0.1395
		[6.94, 20.00]	0.1125	0.2000	0.1000
	⩾ 500	(0.00, 0.52]	0.2000	0.2000	0.1909
		[0.52, 6.94]	0.1750	0.2000	0.1504
		[6.94, 20.00]	0.1625	0.2000	0.1000
B. US	(0, 100]	(0.00, 0.52]	0.1375	0.2000	0.1887
		[0.52, 6.94]	0.1250	0.2000	0.1277
		[6.94, 20.00]	0.1250	0.2000	0.1000
	[100, 1000]	(0.00, 0.52]	0.1875	0.2000	0.1882
		[0.52, 6.94]	0.1875	0.2000	0.1453
		[6.94, 20.00]	0.1750	0.2000	0.1000
	⩾ 1000	(0.00, 0.52]	0.3000	0.2000	0.1900
		[0.52, 6.94]	0.2750	0.2000	0.1534
		[6.94, 20.00]	0.2250	0.2000	0.1000
C. Japanese	(0, 200]	(0.00, 0.52]	0.2250	0.2000	0.1878
		[0.52, 6.94]	0.2000	0.2000	0.1317
		[6.94, 20.00]	0.2000	0.2000	0.1003
	[200, 1000]	(0.00, 0.52]	0.2250	0.2000	0.1905
		[0.52, 6.94]	0.2500	0.2000	0.1370
		[6.94, 20.00]	0.2750	0.2000	0.1005
	⩾ 1000	(0.00, 0.52]	0.4250	0.2000	0.1903
		[0.52, 6.94]	0.4000	0.2000	0.1477
		[6.94, 20.00]	0.5500	0.2000	0.1002
D. European	(0,25]	(0.00, 0.52]	0.1250	0.2000	0.1909
		[0.52, 6.94]	0.1250	0.2000	0.1310
		[6.94, 20.00]	0.1250	0.2000	0.1001
	[25, 200]	(0.00, 0.52]	0.1250	0.2000	0.1932
		[0.52, 6.94]	0.1250	0.2000	0.1438
		[6.94, 20.00]	0.1250	0.2000	0.1001
	⩾ 200	(0.00, 0.52]	0.2000	0.2000	0.1937
		[0.52, 6.94]	0.1750	0.2000	0.1554
		[6.94, 20.00]	0.1750	0.2000	0.1001

Notes. The table contrasts the average asset correlations as calculated in three different ways for the world, US, European, and Japanese portfolios based on asset size and EDF categories. The calibrated average asset correlation is the value reported earlier with respect to the 99.9% percentile. The January correlation proposal value is the constant value of 0.20 implied in the January 2001 proposal by the Basel Committee on Banking Supervision. The November correlation proposal values are the values derived from the formula in the November 2001 proposal by the Basel Committee on Banking Supervision. The formula is presented in the text. The median EDF values for the individual world, US, European, and Japanese portfolios, respectively, are: 0.19, 0.24, 0.26, and 0.19, for the first portfolio; 2.34, 2.57, 2.30, and 2.34, for the second; 20.0, 20.0, 11.450, 15.04, for the third; 0.20, 0.25, 0.20, and 0.14, for the fourth; 1.86, 1.59, 1.99, and 1.65, for the fifth; 16.64, 20.00, 10.80, and 14.25, for the sixth; 0.19, 0.21, 0.215, and 0.13, for the seventh; 1.37, 1.26, 1.48, and 1.18, for the eighth; and 18.61, 20.00, 12.42, and 15.03, for the ninth portfolio.

The results in Table 6 show that the speeds with which the alternative $\bar{\rho}_{\rm A}$ values decline as the EDF values increase are quite different. Within an asset size category, the calibrated $\bar{\rho}_{\rm A}$ values generally either remain constant or decline moderately as the median EDF values increase across the EDF categories, again with the exception of the Japanese portfolios. However, for the regulatory $\bar{\rho}_{\rm A}$ values, the values start near the maximum value of 0.2000 for the lowest EDF category and quickly dive to the minimum value of 0.1000. Thus, by ignoring firm asset size, the regulatory proposal causes greater $\bar{\rho}_{\rm A}$ variability than is suggested by the calibrated $\bar{\rho}_{\rm A}$ values. These results further suggest that a consideration of firm asset size in the calculation of the regulatory $\bar{\rho}_{\rm A}$ values, especially for larger firms, is warranted.

5. Conclusion

The asymptotic single risk factor (ASRF) approach is a simplifying framework for determining regulatory capital charges for credit risk. As described by Gordy (2000a, 2000b, 2002), this framework has become the basis upon which the credit risk capital charges for the second Basel Accord are being determined. In this paper, the ASRF approach was imposed on the KMV methodology for determining credit risk capital charges in order to examine the relationship between average asset correlation, firm probability of default and firm asset size measured by the book value of assets. The analysis was conducted on national portfolios of American, Japanese and European firms, as well as a "world" portfolio composed of these three national portfolios, using data from year-end 2000.

The univariate calibration results indicate that average asset correlation is a decreasing function of the probability of default. This univariate result suggests that the reasons why firms experience rising default probabilities are mainly idiosyncratic and not as closely tied to the general economic environment summarized by the single, common factor. The empirical results further indicate that average asset correlation is increasing in asset size. That is, as firms increase the book value of their assets, they become more correlated with the general economic environment and the common factor. This result is intuitive in the sense that larger firms can generally be viewed as portfolios of smaller firms, and such portfolios would be relatively more sensitive to common risks than to idiosyncratic risks.

The bivariate calibration results support both sets of univariate results and appear to highlight an additional and potentially important relationship between the three variables. The decreasing relationship between average asset correlation and default probability is more pronounced for larger firms. In other words, the average asset correlation for larger firms is more sensitive to firm probability of default than it is for smaller firms. These results provide evidence that both default probability and firm size impact average asset correlation within an ASRF framework, especially for larger firms. Hence, further work regarding whether regulatory capital requirements should take account of other factors seem warranted.

In this paper, we also compared the calibration results with the formula for the average asset correlation proposed by the BCBS in November 2001. This formula explicitly makes the average asset correlation a function of firm probability of default. Based on the regulatory formula, the deviations between the calibrated and regulatory $\bar{\rho}_{\rm A}$ values that are greatest are for the larger firms, suggesting the potential value of incorporating firm size into the regulatory average asset correlation formula. These calibration results provide suggestive evidence that firm size is an important variable in determining credit risk capital requirements and may need to be accounted for in the regulatory calculations within the Basel II process.

Given that several factors must be taken into account in the calibration of the $\bar{\rho}_A$ values used in the ASRF framework, the question arises as to the usefulness of this framework for regulatory capital purposes. Although the ASRF framework is a theoretically simple one to which several credit risk models can be mapped, the cost of simplifying these models down to their ASRF form may be considerable. At the present time, the costs appear to be outweighed by the benefits of having a standard framework to apply across very different financial institutions. Furthermore, these costs will probably be reduced as the BCBS modifies its proposals to account for additional variables, such as firm size, within the ASRF framework, as was most recently proposed in BCBS (2002).

Acknowledgments

The views expressed here are those of the author and not necessarily those of the Federal Reserve Bank of San Francisco or the Federal Reserve System. This work was initiated after several conversations with David Jones from the Federal Reserve Board of Governors, and I gratefully acknowledge his assistance, suggestions and comments. I thank Steven Kealhofer and Jeff Bohn of KMV, LLC for providing me with access to the Portfolio Manager™ software used in the analysis. I also thank Yim Lee, Ashish Das, Kimito Iwamoto, Sherry Kwok, Amnon Levy and Jing Zhang, all from KMV, for their assistance with the data and the software. Finally, I thank Ernst-Ludwig von Thadden (the Editor), two anonymous referees, Akira Ieda, Mark Levonian, Phillip Lowe, George Pennacchi, Marc Saidenberg, and participants at the May 2002 BIS conference on "Basel II: An Economic Assessment" for their comments and suggestions.

References

Asarnow, E., Edwards, D., 1996. Measuring loss on defaulted bank loans: a 24-year study. J. Com. Lending (January), 11–23.

Basel Committee on Banking Supervision (BCBS), 1999. Credit Risk Modeling: Current Practices and Applications. Technical report. Bank for International Settlements.

Basel Committee on Banking Supervision (BCBS), 2001a. The Internal Ratings-Based Approach: Supporting Document to the New Basel Capital Accord. Consultative document. Bank for International Settlements.

Basel Committee on Banking Supervision (BCBS), 2001b. Results of the Second Quantitative Impact Study. Press release (November 5). Bank for International Settlements.

Basel Committee on Banking Supervision (BCBS), 2001c. Potential Modification to the Committee's Proposals. Press release (November 5). Bank for International Settlements.

Basel Committee on Banking Supervision (BCBS), 2002. Quantitative Impact Study 3 Technical Guidance. Technical report. Bank for International Settlements.

Carey, M., 2002. Some evidence on the consistency of banks' internal credit ratings. Manuscript. Board of Governors of the Federal Reserve System.

Carty, L., Lieberman, D., 1996. Defaulted Bank Loan Recoveries. Special Report. Moody's Global Credit Research.

Crosbie, P.J., Bohn, J.R., 2001. Modeling default risk. Manuscript. Moody's KMV LLC. Available at http://www.kmv.com/Knowledge_Base/public/cm/white/Modeling_Default_Risk.pdf.

Das, S.R., Freed, L., Geng, G., Kapadia, N., 2002. Correlated default risk. Manuscript. Leavey School of Business, Santa Clara University.

Gordy, M.B., 2000a. A comparative anatomy of credit risk models. J. Banking Finance 24, 119–149.

Gordy, M.B., 2000b. Credit VAR and risk-bucket capital rules: a reconciliation. In: Proceedings of the 36th Annual Conference on Bank Structure and Competition. Federal Reserve Bank of Chicago, Chicago, pp. 406–415.

Gordy, M.B., 2002. A risk-factor model foundation for ratings-based bank capital rules. FEDS working paper #2002-55. Board of Governors of the Federal Reserve System.

Hamerle, A., Liebig, T., Rosch, D., 2002. Credit risk factor modeling and the Basel II Accord. Manuscript. University of Regensburg.

Hirtle, B., Levonian, M., Saidenberg, M., Walter, S., Wright, D., 2001. Using credit risk models for regulatory capital: issues and options. Fed. Reserve Bank of New York Econ. Pol. Rev. (March), 19–36.

Jackson, P., Perraudin, W., Saporta, V., 2003. Regulatory and 'economic' solvency standards for internationally active banks. J. Banking Finance. In press.

Varotto, S., (2000). Credit risk diversification and bank capital. Manuscript. Birkbeck College.